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Interview with Professor Caroline Series

Caroline started research in ergodic theory but rapidly moved into hyperbolic geometry, developing a special geometrical coding for geodesics with many applications, for which she won a LMS Junior Whitehead prize in 1987. For the last 15 years, she has been working on three-dimensional hyperbolic geometry. She ran a Newton Institute programme on this area in 2003. She is the main organiser of the Warwick Symposium 2006-7 on Low Dimensional Geometry and Topology.

A popular exposition of some of the ideas is to be found in the beautifully illustrated book **Indra's pearls**. *She was born and educated in Oxford and was an undergraduate at Somerville . Having obtained her Ph.D. at Harvard as a Kennedy scholar, she has been at Warwick since 1979. She held an EPSRC Senior Research Fellowship 1999-2004, and went as the LMS Forder Lecturer in New Zealand in 2003. She has served on many committees, both national and international, and is a member of the 2008 RAE Pure Maths panel.*

What led you to becoming a mathematician?

I have always been interested in numbers and patterns since I was very small. I can remember lying in bed at night going through my times tables. The object was to do it without 'repetition, deviation or hesitation'. If I got something wrong I would go back to the beginning and start again. I also remember that I used to pile coloured marbles in pyramids in my bedroom. I enjoyed arithmetic at school. My father (George Series FRS) was a physicist, and I always wanted to study science and understand more about what he was doing. What really got me hooked on mathematics was when we started doing geometry at school. I remember an occasion when I was about twelve when the teacher gave us a geometry problem to take home. I struggled with it for a whole evening and finally managed to do it. Next day at school, I found that nobody else had done it. It was a formative moment. I was so pleased that I had struggled and wrestled with the problem and solved it. I resolved that I would try to solve all the problems we were given. Not long after that I made it my secret ambition to go to Cambridge and get a first in mathematics. At the last minute, I decided to go to Oxford , but other than that, I achieved my goal. My parents were an important influence. My father taught me a fair amount of mathematics. Both my parents felt that education was very important and encouraged me enormously. I went to the Oxford High School for Girls, a GPDST (Girls' Public Day School Trust) school. We had some wonderful teachers; especially I remember our maths teacher Miss Carter.

I was fortunate when I came to do university entrance exams. At that time, you could do entrance exams for both Oxford and Cambridge . The school arranged for a part time teacher called June Chantry, who was a gifted mathematician, to help us prepare. There were two of us to start with but the other girl didn't come back after the first lesson, so I was on my own. Early on, June gave me a lesson to bring me up to speed on determinants. Then she gave me a set of scholarship exam questions to do over the Christmas holidays. This was another situation where I struggled and struggled with the questions but I was not going to give up. I barely knew enough to do them but I managed it. It instilled in me a determination to succeed. She knew exactly the level to pitch something so that it was attainable but only just, at least that is what it felt like to me. Later, she taught Frances Kirwan [1] who went to the same school.

You acquired a first class degree in mathematics at Somerville College, Oxford. Did you enjoy the experience of academic education? Is that when you decided to follow an academic career?

It started with a good interview with the pure maths tutor at Somerville, Anne Cobbe. She asked me a geometry question-something about a Desargian plane. I hazarded a guess and I was right. I became Senior Scholar at Somerville. I found Anne Cobbe immensely inspiring. Sadly, after my first year, she developed cancer and we saw very little of her after that. As you can imagine, the teaching in the college was upset for a while. We had an applied mathematics tutor – Hilary Ockendon – and Jane Bridge (now Kister) who was a logician, who later became an executive editor of Maths Reviews in the USA . They decided to send the pure mathematicians out to other colleges for tutoring and I was given my choice of people to go to. I picked the people whose lectures I had most enjoyed. I went to Peter Neumann, who was enormously influential, to Brian Davies and to Wilson Sutherland. I thrived on all this and was extremely happy at Somerville .

Being in a women's college helped me. There were far more men than women doing maths at Oxford, but in Somerville (at that time it was still a women's college) you had a small community of women around you and you had women tutors. Somehow, I never really felt that I was competing with men. I took it for granted that among them were future geniuses, but that did not concern me. I just worked hard within my own group of women. It was a sort of cocoon but for me it worked really well.

I had not really thought about what I wanted to do beyond my ambition of getting a first, but as the time came closer, I thought I might go into school teaching. I had a boyfriend, another mathematician, who was considered a brilliant student. His tutors encouraged him to go to the United States to graduate school - Michael Atiyah [2] suggested that he should go to Harvard and work with Raoul Bott. The only way I could see to get myself to the USA with him was to go to graduate school too. I discussed this with various people in Somerville , including Hilary Ockendon. Although no one had suggested graduate school before, at that point they were extremely helpful. I ended up winning a Kennedy Scholarship to Harvard so we were able to go together. So I found myself doing a PhD without ever really having thought it through.

How did you get on in the USA and with the PhD? Did it have a big impact on your subsequent career?

I found it quite hard when I first arrived. When you first get there you are relieved that everybody speaks English – it is not as foreign as you thought. Then after a while, you begin to realise that things are more different than you imagined, you go through levels of adjustment. I took a long time to find my feet there. The teaching style was so different. I had been used to tutors who told you what to do; you met them regularly and everything was personal. At Harvard people were kind but it took me an awfully long time to realise that I had to take the initiative about who was going to be my supervisor and what I was going to work on. It sounds ridiculous in retrospect, but it took a year before I made an approach and asked George Mackey [3] if I could work with him. I learnt a huge amount from him about group representations and a wide swathe of mathematics. He was always very supportive and proud that I was his first woman student. So being at Harvard was hard but it was at the same time a wonderful and formative experience. You do not know when you start out whether you are going to be able to write a thesis. In fact, at one point when I was unsure of things, I very nearly switched to a masters degree in statistics, thinking I could always get a job as a statistician.

Being at Harvard had an enormous effect on my life afterwards because you get to know people so well at that time in your life. You saw all the top mathematicians in the world – many were on the faculty and other great people came to give colloquia. Mathematics was passing by in front of you and you were drawn in to its orbit. You also met the other graduate students. Many of them have gone on to have successful careers and are now leading mathematicians, not just in the US but all round the world - there were quite a lot of foreign students. So I have a whole network of people I know really well from those times. When I came back to this country, I knew about the US system and felt completely comfortable there, which was important to me later.

When I finished my PhD, I had a fantastic year in Berkeley where I had my first job. The dynamical systems and ergodic theory people in Berkeley and Stanford included me in their circle. It was just at the end of the hippy era, you had only to walk out on the street in Berkeley and you met all these incredibly strange and interesting people. I made lots of friends who I am still in touch with. It was one of the best years of my life.

Would you say there have been any core themes running through your work?

My thesis, which was really quite obscure, related to ergodic theory. It was about an invention of Mackey's called 'virtual groups'. I realised towards the end of my PhD that what I had done was closely related to a concept called 'orbit equivalence' that Alain Connes [4] had just got interested in, making it a hot topic. I had the good fortune to go to a summer school where Connes was the main speaker. That put me on a good road, but I was not entirely satisfied because what I was doing had become rather abstract. After my year in Berkeley , I came back to Newnham College in Cambridge . I had a research fellowship but I felt isolated mathematically because nobody there seemed to know what ergodic theory was. I decided to broaden my knowledge in dynamical systems and started studying the work of Rufus Bowen who was a professor at Berkeley, one of the leading upcoming people. One evening I suddenly realised that his work on symbolic dynamics, which was very important but quite hard to get your mind round and quite technical, could be illustrated by the example of continued fractions. I also realised that continued fractions provided a prime example of orbit equivalence. Suddenly, I could see all the things I had been trying to get some intuition about illustrated in a beautiful example that related to numbers and to hyperbolic geometry. I then started going into hyperbolic geometry in much more detail. The geometry elegantly interpreted the fascinating patterns involved in continued fractions and the Euclidian algorithm.

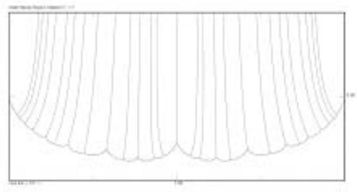
I went back to Berkeley for a couple of summers and I started working on a project with Rufus to extend some of the continued fraction ideas to the wider context of what are called Fuchsian Groups, the symmetry groups of two dimensional hyperbolic geometry. A most dreadful thing happened shortly after we started work – Rufus died suddenly. He had an aneurysm. He was a magnificently strong and healthy Californian in his early thirties. He and his wife were on a hiking trip and he collapsed in a hotel bedroom. I phoned up his home one weekend very excited because I thought I had found a key point in what we were trying to do. Somebody answered and said, "I am very sorry, he is dead". It was a bombshell to us all. I carried on, feeling the best thing I could do was try to finish the project we had started on, which I did. I wrote it up as a joint paper. That set a theme – the connection between continued fractions and hyperbolic geometry – which I went on developing for many years. If you have two periodic motions and the ratio of the periods is irrational, then there is a precise pattern in the sequence of occurrences of the two events. Because the ratio is irrational it is not periodic, but the lack of periodicity is very subtle. There is the same kind of aperiodicity in Penrose Tilings.

Similar patterns are everywhere – they have a lot to do with curves on surfaces.

This is a topic I have kept coming back to throughout my career. You have got complicated curves on a surface and you are trying to simplify them by finding the right kind of moves. It is a bit like the Euclidian algorithm. You have two numbers and you keep simplifying one relative to the other. But now there is no longer a unique way of deciding what to do next. Nowadays people have extended the theory very broadly but I feel we have never quite hit on the 'right' way to do it. If I would like to do one thing, it would be to pin down the most elegant way of expressing these complicated but fundamental patterns.

Have there been special moments of discovery?

One very important moment was when I understood the link between continued fractions, Bowen's symbolic dynamics and orbit equivalence. Another moment occurred much later, when I was collaborating with Linda Keen to interpret some fascinating computer pictures (limit sets of Kleinian groups, see klein.math.okstate.edu/) made by David Mumford [5] and David Wright. By



this time, I had moved from working on two-dimensional hyperbolic geometry into three dimensions. This was all the rage with Thurston [6] making amazing discoveries about hyperbolic geometry and three manifolds. (A Kleinian group is a crystallographic symmetry group of three-dimensional hyperbolic space.) Mumford and Wright had made pictures of a basic example, which again related to continued fractions and curves on a torus. Linda and I were looking in the parameter space of these examples at boundary points that were supposedly transitions into chaos, that is, into non-discrete groups. You could go some way to finding the points because a certain complex parameter had to have the value two. One morning I said 'Why don't we try drawing the lines where the parameter isn't two but where it has real values?' We calculated a bit by hand and it looked interesting. We could not really compute enough by hand, so we contacted David Wright. He came back within a couple of days with an amazing computer picture. The lines where the parameter was real were like rays coming from the inside of the parameter space we were studying and landing on what we thought was the boundary. It was obvious we were on to something. That was a special moment when we saw that picture.

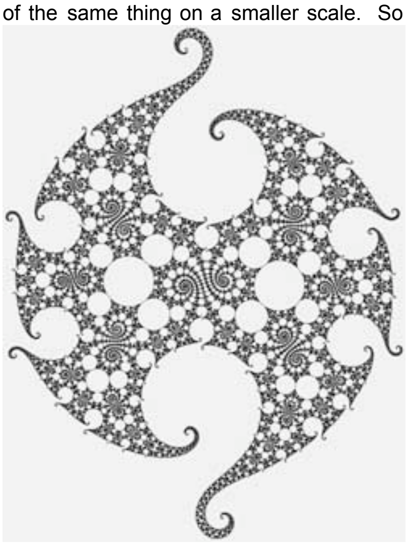
A third moment was after we had spent a year trying to make sense of what the rays meant. One day, I was describing them to Curt McMullen [7] and I asked 'What does it mean, these parameter values being real?' He replied immediately 'It means the convex hull boundary is bent'. (This is a special configuration in three-dimensional hyperbolic geometry.) That turned out to be the explanation of our rays. We had to go from complex analysis into three-dimensional hyperbolic geometry to understand them. From that moment everything just fell into place and we were able to prove that everything was indeed as it looked in the computer pictures.

You are co-author with David Mumford and David Wright of the book *Indra's Pearls* [8] . What is the story behind this collaborative project?

It started with the pictures the two Davids had produced in the mid-seventies. These were the first systematic computer explorations of Kleinian groups. They had many beautiful pictures and decided that they would like to produce a coffee table fractal book rather like 'Beauty of Fractals' by Peitgen and Richter [9] . For a long time, this book project did not get very far. It was the publisher at Cambridge University Press, David Tranah, who kept it on the road. After about ten years of virtually no progress, he suggested to Wright and Mumford, 'Perhaps you need some help.' I knew all about the pictures: I had written a few popular articles about them, and I had been involved in developing the mathematics behind them. So the two things came together and they approached me to join in, which I did. What we planned was not a conventional popular book. We reckoned there was a market aimed at people who have a serious amount of mathematical training such as school teachers, school leavers with A level maths, engineers, undergraduates, computer specialists, etc. We wanted to explain things at a level that people like this could really understand, and we wanted to explain the computing so that they could make their own pictures.

After many delays, we finally got together for a few hours in New York . By then we had started the beginning of the book and David Wright had written a paper related to the ending. Mumford is a great optimist. He said, 'All we've got to do is write the middle to link them together.' Ten years later, the book was complete. It was hard collaborating because we were all in completely different places, thousands of miles apart, but it was great fun. We met up in the middle in Mumford's office in Brown University on Rhode Island . We just sat there and wrote and wrote, drafted and redrafted. I don't think it would have happened if we hadn't had David Tranah pushing us.

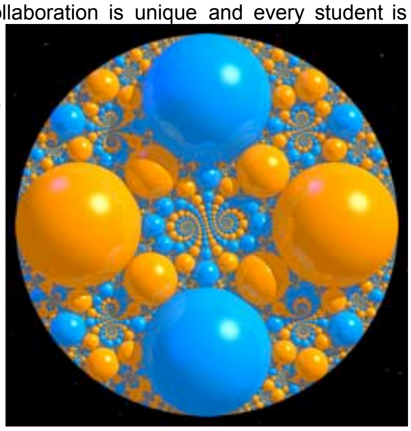
The title [10] is not that of an ordinary mathematics book. I remembered a quotation I had seen in a paper by Mike Berry, about the Hindu myth of Indra's heaven. The idea is that there is a network of silk threads running through this heaven. The silk threads are linked together with pearls; every pearl reflects in another pearl until you see all these reflections within reflections, ad infinitum. This is a metaphor for the idea that within everything there are further repetitions of the same thing on a smaller scale. So it came to my mind that Indra's Pearls would be a good title for the book. During the time we were writing, I began trying to find out where the quote originated. It led me on a journey and a certain amount of detective work. I didn't track down the original source but I discovered that the imagery of things repeating and repeating within themselves is common in certain schools of Buddhism. It is an idea of infinity very different from the Western way of counting 1,2,3,4,5… , but it seems apt for this kind of fractal repetition. As we might say: 'In each sack were seven cats and each cat had seven kits…'. Philosophically, it seemed a better way of thinking about infinity for the kind of mathematics we wanted to describe.



It was our hope that amateurs would take up this challenge of making their own pictures. We have been delighted that a lot of people have. Based on our instructions, a mathematics graphics artist called Jos Leys has made magnificent versions of the more elaborate pictures in our book. He has tweaked them and made them artistic, a sort of heavenly universe. You can see them on his website http://www.josleys.com.

You have had numerous collaborations with men and women. You have also taught a number of women students. Have you ever noticed differences in working with men and women?

No, I haven't noticed any particular differences, you have to take everyone as an individual and take it from there. Every collaboration is unique and every student is different from every other. I have had two particularly long and productive collaborations with women. One was with Joan Birman and the second was with Linda Keen. Both are American, so we did our work while staying in each other's homes for quite long periods. I had lots of fun with both of them, learning about new kinds of cooking, gossiping, and sharing many other things. I am sure that men have equally close collaborations, but it would have been difficult to work with a man in this way. But I have also had enjoyable collaborations with men. I find now that I am working with younger people, it is harder to develop the same degree of closeness. It tends to be more formal, you meet in your office and go home and work on your own.



You were one of the founders of the organisation European Women in Mathematics [11] . What was the intention behind this and what has it achieved so far?

I first encountered an organised group of women mathematicians when as a graduate student I was invited to a meeting of the Boston Women in Mathematics. It was so warm and pleasant to be sitting tightly packed on the floor in somebody's house, all women involved in mathematics. After that, we set up a little group of the women graduate students at Harvard and I made some close friends. I did not think much more about it until the International Mathematical Congress in 1986 in Berkeley . The Association of Women in Mathematics, the AWM, which had existed in the US for some years, invited women from different countries to speak in a panel discussion. There were a number of us Europeans there, all belonging to the American organisation. It came to me that we ought to be doing this in Europe . I floated the idea and we decided to try. A small group of us set up a first meeting in Paris. Then we had a meeting in Denmark the following year. The third year, I offered to organise a meeting in Warwick . It was all tremendously exciting. We were making things up 'on the hoof' and we had to think ahead. If you did not have a suitable plan of action prepared, the meeting would happen and everyone would go away and it would all fizzle out. There was no funding. It was a hand to mouth existence, but it grew. We set ourselves up as a legally constituted organisation before the European Mathematical Society and gradually we tapped into various sources of funding. It is not a big organisation but we hold meetings every couple of years somewhere in Europe .

EWM offers the kind of friendships and networking that evolve largely through working together and making the organisation function – meeting women of all different ages and hearing their stories. I hope that we have been a support to younger women.

They see women who have made it. Perhaps one or two have been at a difficult point in their career and we have convinced them to carry on. In the UK we now have the LMS Women in Mathematics Committee, which had its roots in all this activity. This year, 2007, we are hosting the 13th EWM General Meeting in Cambridge [12] . Again I have found myself on the Organising Committee, though playing a much smaller role than the first time around.

I understand you were behind setting up your university environment committee. How did you do this and what led to you being involved in it?

For a long time I have had a growing concern that we do not pay enough attention to environmental matters in everyday life. This particular enterprise began because I was trying to find out why the informal paper recycling system in our department suddenly ceased to exist. This led me gradually higher up a chain of consequences and I began to realise that it was a problem much larger than a little bit of paper recycling. The University did not have any niche in its administrative structure to consider environmental questions. About the time this was crystallising in my mind, there was an opportunity to write an article in a professorial newsletter. I had a response from one colleague, Professor Mike Terry in the business school. We gathered people together and set up an informal group, the first forum to which people could bring environmental issues. The Registrar was very supportive. However our group had no resources and we had no official existence even though we had the Registrar as a member. After about five years, it became an official university committee under a new Registrar's chairmanship. Just under a year ago, the University appointed an environmental officer, something our group had been pressing for years.

The University is now involved in a project with the Carbon Trust (www.carbontrust.co.uk) and is making great efforts to reduce its carbon emissions. These issues are tremendously important. It is not that people are unwilling to help but you need the infrastructure to provide a vehicle. That was really what I wanted to see happen. At Warwick it needs more work but we have made a good beginning. I am glad these ideas are finally reaching their time.

Besides research, what elements of your career do you find most rewarding?

I have enjoyed many aspects of my career, but everything always revolves around the core of the research. If you didn't have research at the heart of it, the other things would seem less meaningful. I think the most important thing after that is just wonderful colleagues. One has all sorts of different networks. I am very lucky to be in a supportive, friendly department in Warwick . I felt from the first moment I started there that it was an inclusive place where everyone's opinion counted. Then you have a national network. I have been quite involved in the London Mathematical Society and so on. Then you have people all around the world in your field of research who you meet from time to time. However different your background, you have your underlying interest in mathematics, so you can immediately relate. You meet people from so many different places. You get the chance to travel and meet them in their own countries. Sometimes they invite you to stay in their homes. There are always new people, interesting young people coming along, undergraduates, postdocs, and so on. There is never a dull moment. It is all truly rewarding.

Terry Edwards

Aknowledgement Colour illustrations courtesy of Jos Leys http://www.josleys.com

[1] Frances Kirwan FRS . President of the LMS 2003-4.

[2] Sir Michael Atiyah FRS . Fields medal winner 1966

[3] George W. Mackey 1916-2006, FAAS. See forthcoming article in the Notices of the AMS , 2007.

[4] Alain Connes, Fields medal winner 1982

[5] David Mumford, Fields medal winner 1974

[6] William Paul Thurston, Fields medal winner 1982

[7] Curtis McMullen, Fields medal winner 1998

[8] klein.math.okstate.edu/

[9] Springer-Verlag, 1986, ISBN 0-387-15851-0

[10] Indra's Pearls: The Vision of Felix Klein, Cambridge University Press, ISBN-13: 9780521352536

[11] http://www.math.helsinki.fi/EWM/

[12] http://www.maths.cam.ac.uk/ewm/